Application of Airfoil Theory for Nonuniform Streams to Wing Propeller Interaction

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The effect of a single propeller on the aerodynamic characteristics of a high aspect ratio wing has been studied. The basic assumptions of classical analysis have been retained except the one for the two-dimensional sectional analysis. For the section inside the propeller stream, classical analysis assumes that the sectional height of the propeller stream is infinitely large. When the oncoming propeller stream is uniform and with a distinct boundary with the outer stream the modification is now made to take into consideration the finite height of the higher velocity propeller stream in the calculation of the sectional lift. When the oncoming propeller stream is not uniform, and without distinct boundary, the sectional two-dimensional analysis is computed numerically by a finite-difference scheme, while in the three-dimensional analysis of downwash, the propeller stream is replaced by an equivalent circular cylindrical jet of uniform velocity. The numerical examples show that the modifications due to the use of more realistic two-dimensional analysis are significant.

Nomenclature

C	= coefficient determined by $(S^2-1)/(1+S^2)$
C_L	= sectional lift coefficient
C_{Lo}	= sectional lift coefficient due to wing alone
c	= chord
c_o	= maximum chord
D	= coefficient determined by $(S-1)^2/(1+S^2)$
$dC_L/d\alpha$	= sectional lift curve slope
L	= total lift curve slope
l	= sectional lift
l_o	= sectional lift due to wing alone
l_p	= net sectional propeller lift
r_j	= radius of the propeller
S	= velocity ratio, u_j/u_o
s	= half wing span
$\bar{s}$	= dimensionless half wing span, s/r_j
u	= upstream velocity
u_j	= propeller stream velocity
u_o	= freestream velocity
w	= sectional downwash
w_j	= downwash for wing section inside the propeller stream
w_o	= downwash for wing section outside the propeller stream
y	= spanwise coordinate
$\bar{y}$	= dimensionless spanwise coordinate, y/r_j
α_g	= geometric angle of attack
α_o	= the angle of zero lift
Γ	= sectional circulation
Γ_o	= sectional circulation due to wing alone

I. Introduction

IN the classical theory for wing-propeller interaction due to Koening[1], the stream velocity in the section lift analysis is taken to be uniform u_j for the section inside the propeller stream and to be u_o for the section outside of it. In general, the velocity profile behind the propeller is nonuniform. In the literature,[2-5] it was shown that the lift of a two-dimensional airfoil is sensitive to upstream nonuniformities in a manner which substantially affects the aerodynamic characteristics of the wing section. Among the preceding references Vidal[2] deals with an oncoming flow of constant shear, Ting and Liu[3] deal with step velocity profile, Chow et al.[4] treat the flow of a general nonuniform profile and Ludwig and Erickson[5] use a composite uniform shear profile.

If the wing-propeller interaction is considered under the general scheme of Prandtl, whereby a large scale three-dimensional problem is coupled with a small scale local two-dimensional problem, the nonuniformities in the velocity profile generated by the propeller may have significant effects in the determination of the sectional lift distribution over the wing. It is clear that even under the idealized conditions considered by Koening,[1] from the local two-dimensional point of view, as one proceeds from the center of the propeller flow to the side, the oncoming stream exhibits a diminishing width of propeller air flow (Fig. 1). It is the purpose of this investigation to establish the effect of the aforementioned nonuniformities on the lift distribution over the wing by taking into consideration the accurate solution of the two-dimensional problem. This improvement is also consistent with the lifting line theory. It is well-known that lifting line theory holds not only for a thin airfoil at small angle of attack[6] but also for a thick airfoil at finite angle of attack.[7] For the latter case the sectional lift has to be obtained by nonlinear analysis, for example, conformal mapping. Under lifting line theory the flow deflection near the wing section at a distance of the order of the chord can be finite. However, at a distance of the order of the span, the flow deflection is small. In the same spirit, the improved sectional analysis

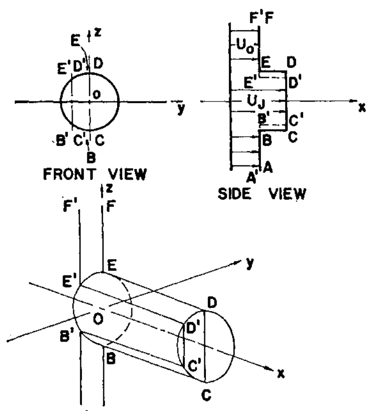

Fig. 1 Decrease in propeller width in wing direction.

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for an airfoil in nonuniform streams[4] can be applied for airfoils at large flap angles. On the other hand, in the three-dimensional analysis of the lifting line and the cylindrical stream behind the propeller, where the length scale is of the order of the span, the assumption of the small deflection angle is valid.

Attempts at improving the classical analysis were concentrated on replacing the lifting line by the lifting surface with the Weissinger approximation.[8] The justification of this approximation is that it yields the value 2π for the lift slope of a two-dimensional airfoil. Jameson[9] has extended Rethorst's lifting surface theory for jets with a circular cross section to a rectangular and an elliptic cross section. Although the results to be presented here are based on a modification of the lifting line theory for the wing-propeller interaction, it is expected that the contribution due to these nonuniformities would remain proportionately the same whether lifting line or a more elaborate lifting surface theory is employed.

Recently a systematic asymptotic expansion scheme[10] has been developed for the analysis of the interference of the wing with multi-propellers. In the analysis it becomes clear that, under the assumption of a large aspect ratio, the modification in the sectional lift is the leading correction while the improvement of lifting line theory by the lifting surface theory is of high order.

In Sec. II, the general equations and method of analysis are briefly presented. A detailed account of the underlying assumptions can be found in Ref. 11. A comparison between the present analysis and an available calculation due to Koening[12] for an infinite wing is presented. In Sec. III, the effect of nonuniformities is considered for finite wings. The analysis is carried out for an upstream flow consisting of a step function velocity profile.The contributions due to nonuniformities are determined by comparing a calculation using uniform values for α_0 and $dc_L/d\alpha$ with a calculation based on variable values of these coefficients as determined by the analysis. It is shown that, in the examples considered, the classical results overestimate the net lift due to the propeller by up to 40%. When the freestream consists of a nonuniform smooth velocity profile, i.e., with no jump discontinuities at the propeller boundary, a solution is determined by defining an equivalent step profile. Results are presented for an elliptical planform wing using a flat plate and Joukowski airfoil sections at various angles of attack. By using the appropriate sectional lift coefficient from Refs. 3 or 4 as input, the numerical computation for each interference problem requires about two minutes on the CDC 6600.

II. System of Equations and Method of Solution

The lift line theory for the wing-propeller interaction as presented in Ref. 3 consists of two basic equations: the distribution of circulation and the distribution of downwash. With minor changes in notation, the results are summarized below.

The distribution of circulation,

$$\Gamma(y) = \tfrac{1}{2}\,(dC_L/d\alpha)\; uc\;[(\alpha_g - \alpha_0) - w/u] \tag{1}$$

where $\Gamma(y)$ is the circulation at y, u is the upstream velocity, $c(y)$ is the chord distribution, α_0 is the zero lift angle, α_g the geometric angle of attack and (w/u) is the induced angle due to downwash.

The downwash equation

$$w_J = \frac{1}{4\pi}\left\{(1-D)\left(\int_{-\bar{s}}^{-1} + \int_{1}^{\bar{s}}\right)\frac{d\Gamma}{\bar{y}-\eta} + \int_{-1}^{1}\frac{d\Gamma}{\bar{y}-\eta} - C\left(\int_{-\bar{s}}^{-1} + \int_{1}^{\bar{s}}\right)\frac{d\Gamma}{\bar{y}-\dfrac{1}{\eta}}\right\} \tag{2a}$$

valid for $|\bar{y}| < 1$, and

$$w_0 = \frac{1}{4\eta}\left\{\left(\int_{-\bar{s}}^{-1} + \int_{1}^{\bar{s}}\right)\frac{d\Gamma}{\bar{y}-\eta} + (1-D)\int_{-1}^{1}\frac{d\Gamma}{\bar{y}-\eta} + C\left(\int_{-\bar{s}}^{-1} + \int_{1}^{\bar{s}}\right)\frac{d\Gamma}{\bar{y}-\dfrac{1}{\eta}}\right\} \tag{2b}$$

valid for $1 < |\bar{y}| < \bar{s}$.

In Eq. (2), w is the downwash, $\bar{s}$ is the half wing span, and the coefficients C and D are given by:

$$C = (S^2 - 1)/(1 + S^2) \quad \text{and} \quad D = (S - 1)^2/(1 + S^2) \tag{3}$$

where $S = u_J/u_0$ with u_J designating the velocity over the propeller part and u_0 the freestream velocity. Distances such as $\bar{y}$, η, and $\bar{s}$ have been nondimensionalized with respect to the propeller radius.

In the classical setting, the preceding system of equations is complete once the values of the coefficients $dC_L/d\alpha$ and α_0, the wing characteristics at uniform flow, and $c = c(y)$, the chord distribution have been prescribed as supplementary information. The values of the geometrical angle of attack and the propeller to freestream velocity ratio S are the parameters in the problem.

In the present analysis, the preceding system is solved in conjunction with a supplementary program developed in Refs. 3 and 4. In these programs, for any given upstream velocity distribution and a wing section at a specified geometric angle of attack the aerodynamic coefficients α_0, $dC_L/d\alpha$ are determined. With the wing characteristics known as function of y, the system of equations, Eq. (1) and Eq. (2), are solved by the method of colocation.

As a check on the present numerical procedure, in Fig. 2, a comparison is shown with a calculation due to Koening.[12] The calculations of Ref. 12 are based on a perturbation technique which has been applied to the interaction of a propeller and a wing of infinite span. To simulate an infinite span in a program designed for a finite wing, calculations have been carried out for span-to-chord ratios of 24, 36, and 48, i.e., $\bar{s} = 12$, 18 and 24, respectively. As can be seen from Fig. 2,

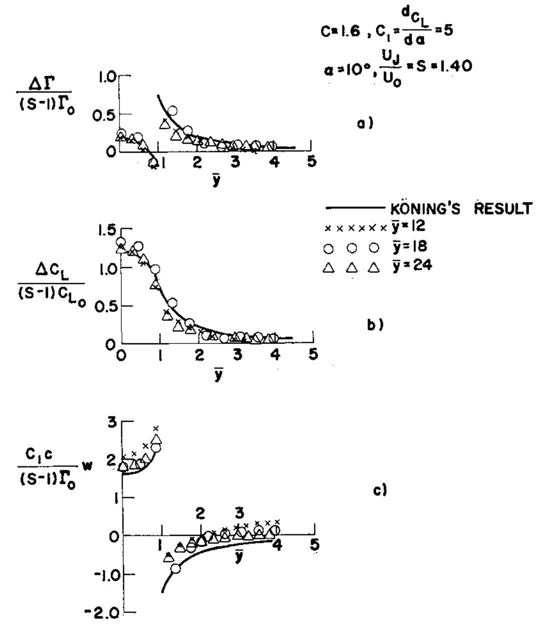

Fig. 2 Propeller wing interaction for a wing of infinite span a) change in circulation, b) change in lift, and c) downwash.

good agreement is obtained for circulation and lift already at $\bar{s} = 18$. Results for the downwash indicate a slower rate of convergence to the $s \to \infty$ case.

Numerical results are presented for

1) The circulation Γ in terms of the variable

$$\Delta\Gamma/(S-1)\Gamma_0 = \Gamma - \Gamma_0/(S-1)\Gamma_0 \quad (4)$$

where Γ is the total circulation and Γ_0 is the circulation due to the wing alone.

2) The net propeller lift $l_p(y)$ in terms of the difference in the lift coefficients

$$\Delta C_L/(S-1)C_{L_o} = (C_L - C_{L_o})/(S-1)C_{L_o} \quad (5)$$

where

$$C_L = l(y)/\tfrac{1}{2}\rho u_0{}^2 c_0 \quad (6)$$

with c_0 the maximum chord and C_{L_o} the lift coefficient due to the wing alone (i.e., no propeller), and

3) The downwash in terms of the nondimensional quantity

$$w' = \frac{(dC_L/d\alpha)c_0}{(S-1)\Gamma_0} w \quad (7)$$

In subsequent sections, a comparison between the lift distribution based on uniform aerodynamic coefficients and the lift distribution as obtained by taking nonuniformities into consideration will be presented for wings of finite span.

III. Step Velocity Profile

The effect of nonuniformities in the interaction of a propeller and a finite wing is investigated here using the step velocity profile defined by

$$u = \begin{cases} u_j = Su_0 & 0 \le |y| < r_j \\ u_0 & |y| = r_j \end{cases} \quad (8)$$

In Eq. (8), y is a dimensional quantity and r_j is the propeller radius.

In order to isolate the effect of α_0 and $dC_L/d\alpha$, a flat plate wing section is considered first. The values of α_0 are thus zero for all points on the wing. Using an elliptical plane form, the interaction problem is considered subject to variation in the following parameters: span-to-propeller ratio $2s/r_j = 2\bar{s}$ velocity ratio $S = u_j/u_o$; and geometrical angle-of-attack α_g.

With the fixed chord-to-propeller ratio $c_o/r_j = 1.37$ used in the present calculations, a value which corresponds to a conventional airplane, variation in span-to-propeller ratio $(2s/r_j)$ becomes equivalent to variation in span-to-chord ratio $2s/c$.

The range variation in the parameters are shown in Table 1.

In Fig. 3, the sectional lift distribution for *FP*1 is shown in terms of the lift coefficient C_L [Eq. (6)]. The maximum difference between the classical and the present results is exhibited over the propeller region. The two distributions approach one another rapidly as $|y|$ becomes longer than r_j. This fast convergence is an immediate consequence of the fact that the aerodynamic coefficients are identical in the two sets of calculations for all $|y| > r_j$. The net propeller lift distribution $l_p(y)$ which is obtained by subtracting the lift of the wind alone $l_o(y)$ from the sectional lift $l(y)$ is shown in Fig. 4. It can be seen that the total propeller lift is defined by

$$L_p = \int_{-\bar{s}}^{\bar{s}} l_p(y')\, dy' \quad (9)$$

is overestimated by close to 20% in this calculation when the nonuniform corrections of the present theory are ignored.

In Fig. 5, these calculations are repeated for the *FP*2 case where the span-to-propeller ratio has been doubled. Since the contribution to the total lift $L = \int_{-\bar{s}}^{\bar{s}} l(y)\, dy$ due to wing increases as s/r_j increases the overall effect due to the present correction becomes proportionately smaller. On the other hand, the effect on propeller lift as shown in Fig. 6 again shows an overestimate of the classical calculations by 20%.

It is expected that more drastic variations between the two calculations, classical and present theory, will be exhibited as a consequence of an increase in $S = u_j/u_o$ or an increase in the

Table 1 Range of variation of parameters for flat plate section ($c_o/r_j = 1.37$)

Case	$\bar{s} = s/r_j$	$S = u_j/u_o$	α_g
*FP*1	3.5	2.0	5
*FP*2	7	2.0	5
*FP*3	3.5	2.5	5
*FP*4	3.5	2.0	7.5
*FP*5	7	2.5	7.5

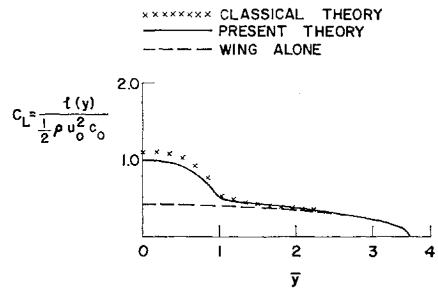

Fig. 3 Sectional lift distribution—*FP*1.

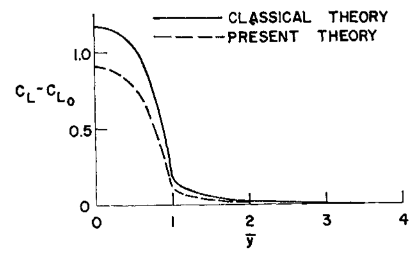

Fig. 4 Net propeller lift distribution—*FP*1.

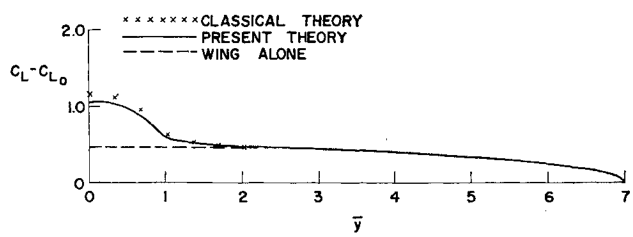

Fig. 5 Section lift distribution—*FP*2.

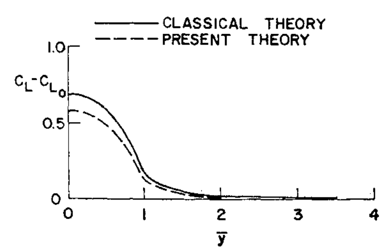

Fig. 6 Net propeller lift distribution—*FP*2.

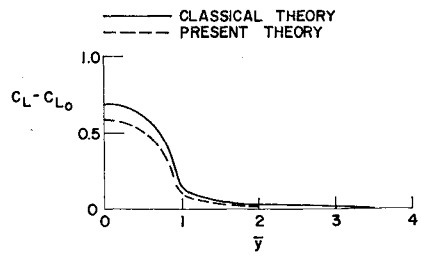

Fig. 7 Net propeller lift distribution—*FP*3.

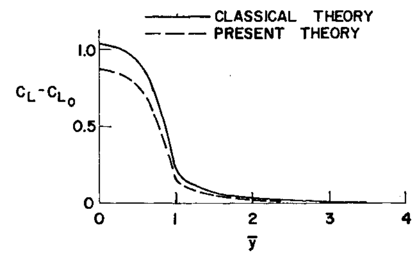

Fig. 8 Net propeller lift distribution—*FP*4.

angle of attack α_g. In Fig. 7, a comparison is made for the l_p distributions using the *FP*3 which is the same as *FP*1 except that the velocity ratio was increased from 2 to 2.5. The overestimate in net lift by the classical theory reaches here a value of 30%. In Fig. 8, the effect of angle of attack is considered by using the *FP*4 case. Variations for the *FP*4 case reach a value of 25%.

As the last calculation for the flat plate case, the data for *FP*5, in which all the upper values of the parameters have been used, has been considered. The results for the net propeller lift are compared in Fig. 9 where the variation in net lift when nonuniformities are included or not exceed the value of 40%.

With the preceding results, it is clear that once one moves away from a perturbation condition such as $S = u_j/u_o \sim 1$, and small geometrical angles of attack, the two-dimensional corrections as proposed in this report become essential. High angles of attack and large u_j/u_o values are standard operational conditions for VTOL/STOL.

In order to evaluate the effects of both α_0 and $dC_L/d\alpha$, a calculation was carried out for an elliptical plan form with a Joukowski airfoil. The parameters considered are shown in Table 2.

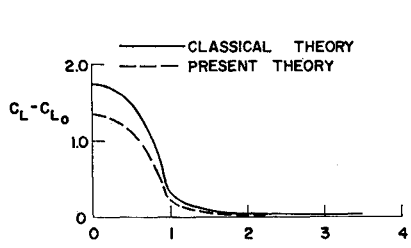

Fig. 9 Net propeller lift distribution—*FP*5.

Table 2 Parametric values for Joukowski airfoil

Case	$\bar{s} = s/r_j$	$S = u_j/u_o$	α_g
*JA*1	3.5	2	0
*JA*2	3.5	2	4
*JA*3	3.5	2	6

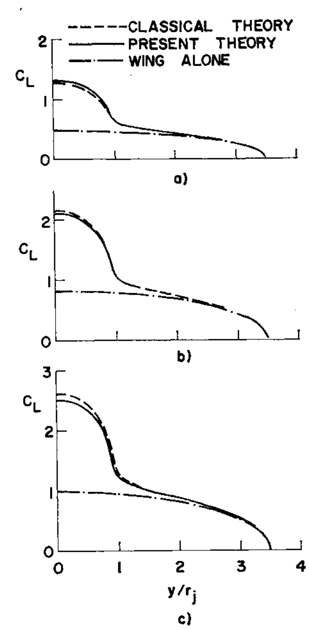

Fig. 10 Sectional lift distribution for Joukowski airfoil a) *JA*1, b) *JA*2, and c) *JA*3.

In Fig. 10, parts *a*, *b*, and *c* results are shown for the *JA*1, *JA*2, and *JA*3 cases respectively. As can be observed from the figures, the variation between the classical and present analyses changes its direction as the value of α_g changes. It appears that α_g has a compensating role to the $dC_L/d\alpha$ effect which fortuitously may decrease the deviation between the two calculations as shown in the *JA*2 case. Such occurances cannot be anticipated a priori and therefore calculations including the distributed nature of α_o and $dC_L/d\alpha$ have to be carried out.

IV. Nonuniform Smooth Velocity Profiles

In a real situation when the propeller is located in front of the wing, the oncoming stream does not possess a discontinuous velocity profile but a smooth one. When the propeller is centrally located, to a good approximation, the profile has an axial line of symmetry. Within the framework of lifting line theory, a solution for the interaction problem is obtained by relating the circulation equation

$$\Gamma(y) = \tfrac{1}{2}(dC_L/d\alpha)u(y)c(y)[\alpha_g - \alpha_0) - (w/u)] \quad (10)$$

which constitutes the local two-dimensional picture to the downwash expression (w/u) which in turn represents the global three-dimensional picture. For example, when there is a single jump in the velocity profile, the downwash expression as given explicitly by Eqs. (2a) and (2b) involves the coefficients C and D which are related to the jump through the parameter $S = u_j/u_o$.

When the velocity profile is smooth, Eq. (10) remains unaltered. With the aid of the program of Ref. 4, the values of α_o and $dC_L/d\alpha$ are determined for each point y on the profile. Furthermore, with this program the appropriate value of the velocity associated with the streamline which passes through the trailing edge in accordance with the Kutta condition is also determined. It is this value of the velocity $u = u(y)$ which must be used in Eq. (10).

In order to complete the problem, an appropriate representation for the downwash is required. In principle, such an expression could be generated by dividing the uniform profile into a finite number of steps and then apply the interface conditions of continuity of pressure and streamline direction between any two consecutive concentric fluid cylinders. An inspection of Eq. (2) which has been determined for a single step (i.e., a single interface condition) reveals that such an extension is algebraically complex. In order to simplify the calculations, an "equivalent step" method is proposed.

In this approximation, the local two-dimensional picture [i.e., Eq. (10)] and the pertinent aerodynamic coefficients as determined for the nonuniform profile by the program of Ref. 4, are retained. The downwash expression, however, is approximated by replacing the nonuniform profile by one with a single step in the velocity such that two integral properties of the step profile are equal to the corresponding two integral properties of the nonuniform profile. This equivalence is made utilizing the two degrees-of-freedom available, i.e., the size of the propeller and the magnitude of the propeller flow velocity.

The first integral property chosen is the momentum defect which is invariant under diffusion. Thus, if $u(y/c_o)$ is the nonuniform profile, specified as a function of y/c_o where c_o is the maximum chord, the momentum defect is given by

$$\int_0^\infty \left(\frac{u}{u_o}-1\right)\left(\frac{u}{u_o}\right)\left(\frac{y}{c_o}\right)d\left(\frac{y}{c_o}\right)=\frac{1}{2}(S-1)S\left(\frac{r_J}{c_o}\right)^2 \qquad (11)$$

Since there are only two degrees-of-freedom, the choice of the second integral is necessarily not unique. Maintaining, however, that the lowest order moments are the most significant ones, the second integral property is chosen to be the displacement thickness viz,

$$\int_0^\infty \left(\frac{u}{u_o}-1\right)\left(\frac{y}{c_o}\right)\mathrm{d}\left(\frac{y}{c_o}\right)=\frac{1}{2}(S-1)\left(\frac{r_J}{c_o}\right)^2 \qquad (12)$$

If a representative nonuniform profile is chosen in the form

$$u(y/c_o)=u_o\{1+a\exp[-(y/c_o)^2/(b/c_o)^2]\} \qquad (13)$$

applying Eqs. (11) and (12) to this profile yields after solving for S and r_j/c_o,

$$(S)_1=1+a/2, \quad (r_j/c_o)_1=(2)^{1/2}(b/c_o) \qquad (14)$$

As an example, an elliptic wing with an aspect ratio of 6.5 and a Joukowski wing sections at an angle of attack $\alpha_g=5°$ was considered with nonuniform flow given by Eq. (13) with $a=1$ and $b/c_o=0.55$. The equivalent step interaction problem as determined by Eq. (14) has the parameters,

$$(S)_1=1.5, \ (r_j/c_o)_1=0.78$$

By replacing the second integral property [Eq. (12)] with the energy defect,

$$\int_0^\infty \left(\frac{u^2}{u_o{}^2}-1\right)\left(\frac{u}{u_o}\right)\left(\frac{y}{c_o}\right)d\left(\frac{y}{c_o}\right)=\frac{1}{2}(S^2-1)\,S\left(\frac{r_J}{c_o}\right)^2 \qquad (15)$$

instead of the system Eq. (14), one obtains for the parameters S and r_j/c the expressions

$$(S)_2=\tfrac{2}{3}\,(a^2+3a+3)/(a+2)$$

and (16)

$$(r_j/c_o)_2=\tfrac{3}{2}(b/c_o)(a+2)[(a+2)/(2a+3)(a^2+3a+3)]^{1/2}$$

With $a=1$ and $b/c_o=0.55$, the equivalent step interaction problem has the parameters,

$$(S)_2=1.556, \ (r_j/c_o)_2=0.726$$

i.e., not substantially different from the values obtained based on the displacement thickness.

The lift distributions as generated by the preceding two sets of parameters are shown in Fig. 11 as distribution l_1 and l_2, respectively. As can be seen from the preceding figure, the difference between the total lift obtained in the two calculations viz. $(L_1-L_2)/L_1$ is less than 3%. It appears that when equivalence is defined by using the lowest moments, the resulting lift distribution is not too sensitive to the second integral condition used.

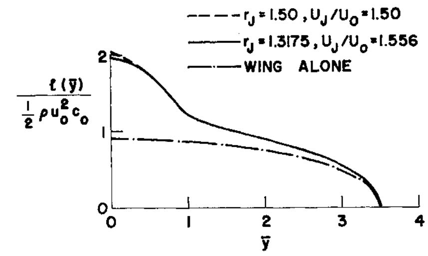

Fig. 11 Sectional lift distributions—equivalent jet.

V. Conclusions

The effect of nonuniformities in aerodynamic coefficients on the lift distribution has been considered by using a simple extension of lifting line theory. For uniform velocity profiles using a flat plate wing section, it was shown that ignoring nonuniformities could lead to overestimates in lift up to 40%. When Joukowski profiles were considered, it was shown that α_o and $dC_L/d\alpha$ may have an opposite effect on the results thus leading in some cases to overestimates and in other cases to underestimates in lift when compared with classical calculations.

For nonuniform velocity profile, in order to avoid a highly complex analysis, an approximation has been proposed whereby an equivalent step profile is replacing the nonuniform one. Equivalence has been defined by imposing the conditions that the momentum defect plus one other integral property should be equal for the two profiles. It was shown that when the second integral property is a low moment, such as the displacement thickness or the energy defect, the resulting lift distribution is practically the same.

While, within the existing theory, the importance of a proper account of nonuniformities has been clearly demonstrated, in order to verify how accurately the proposed theory represents the physical situations, additional experimental and analytical studies are desirable.

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